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# Cognitive Dynamics: Additive or Multiplicative?

*Original Research*

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## Abstract

**Introduction**: Cognition is assumed to rely on distinct and additive substages such as perceptual encoding, memory, and motor control. Nevertheless, questions surrounding the assumptions of modularity and additivity persist. If a stable cognitive architecture exists, then repeatedly executing the same cognitive act should repeatedly engage the self-same structure. If discreet sub-acts behave in a manner consistent with a sum of independent random variables, then the assumption of additive and modular cognitive processes is reasonable. However, if they develop dependencies, then the assumption of additivity and modularity in cognition should be questioned.
**Methods:** The study required participants (N = 180) to successively execute identical elementary cognitive acts in a stacked 1-word, 2-word, and 4-word lexical decision task. Correct response time was the primary dependent measure.
**Results**: Statistical analyses revealed evidence for additivity in mean response time after a logarithmic transformation ($r^2 = .81, p < .05$ & $r^2 = .74, p < .05$). This pattern is consistent with multiplicative dynamics.
**Conclusions**: The results indicate that variance grows multiplicatively as a function of the number of sub-acts. A straightforward way to generate this pattern of variability growth is to assume the sub-acts develop successive dependencies and combine multiplicatively.

***Key Words***: additive factors, multiplicative interactions, nonlinear dynamics.

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## Introduction

### *Additive Dynamics*

Additive dynamics refer to patterns of change for which causes are independent and combine additively such that outcomes reduce to a simple sum of inputs, plus Gaussian noise. For example, in a head-on collision, two identically forced billiard balls will simultaneously halt since equal and opposite forces sum to cancel. Multiplicative dynamics refer to patterns of change in which effects and noise are amplified or attenuated in proportion to the influence of previous effects and states. For instance, the amount credited by a fixed interest rate is proportionally dependent on an account's balance. The same interest rate yields small credits for small balances, and large credits for large balances.

Considerable scientific effort is directed at delineating the structural components of the mind. Brain-focused methods rely on subtractive logics, such as *Region of Interest* fMRI and PET scans.[1] Since all subtracted brain images must entail points of minimum and maximum difference, only an *a priori* true theory could unambiguously allow one to distinguish spurious from real differences.[2,3,4] Saul Sternberg's (1969) Additive Factors method relies on response time measurements and is perhaps the most widely subscribed behavioral technique for localizing cognitive mechanisms.[5] Independent variables, or factors, thought to be associated or dissociated with a given cognitive faculty are manipulated factorally in ANOVA designs. If one factor's influence on mean response time is additive with another, then they impact distinct components; if the factors interact, they impact a common mechanism. Falsifiability is a key advantage of the additive-factors method.[6]

So far, no cognitive mechanism has been successfully individuated to the point that consensus exists among cognitive scientists about its functional and empirical details.[8,9] Perhaps hypothesized cognitive factors are somehow incongruous with true brain functions, or perhaps the working hypothesis of dedicated mechanisms supporting cognition should be revisited. The upshot is persistent ambiguity surrounding modularity hypotheses using standard factorial or subtractive research designs.

By 1810, P.-S. Laplace realized that sums of independent sources of unsystematic deviation produce a pattern of variability tending to a Gaussian probability density function. This observation, along with earlier work by de Moivre, was formalized as the Central Limit Theorem (CLT).[7] The CLT is the backbone of linear statistical analysis. Summing independent observations, as when a mean is computed, tamps down the influence of extreme values. If the sources of variability are unsystematic and additive, the result is a symmetric Gaussian distribution. The sampled population distributions could be anything from a tightly bounded rectangular distribution to a skewed exponential distribution. This is true even for more complex distributions like the ex-Gaussian or ex-Wald. To be pertinent, the CLT only requires the samples be independent, sufficiently large, and drawn from populations with a finite means and variances.

Rather than seeking to individuate cognitive mechanisms, this article tests if discrete, independent, and complete cognitive acts, when combined, behave as truly isolable cognitive acts. If the durations of sequences of distinct cognitive acts behave as simple additive sums of the individual acts, then hope remains for modular hypotheses. Otherwise, the organizing principle of modularity should be reconsidered.

*Multiplicative Dynamics*

Several additional statistical facts are required for the planned test. They concern the expected variance of sums and products of independent random variables. The variance of a sum or a difference of two independent random variables is the sum of the variances of the combined variables. Thus, as independent random variables are combined in successive sums or differences, their variance grows additively, and their distributions become increasingly Gaussian. However, if the variables are correlated, negatively or positively, the aggregated variables will undershoot or overshoot, respectively, the variance expectations of independent variables.

The expected variance of products of independent random variables are the products of the sums of each variable's variance and the square of its mean, minus the products of the means, squared. The upshot is the variance of products of independent random variables vastly outpaces the variance of sums of independent variables. Next, we report a lexical decision experiment designed to leverage the statistical power of the CLT and the variance rules to contrast the traditional hypothesis that cognitive acts combine additively with a more-than-additive combination hypothesis, such as positive correlation, or multiplicative interaction.

Three different between-subjects lexical decisions conditions impose a progressive doubling of the number of successive individual lexical decisions participants must complete on each trial. The single item, or 1-word condition uses a standard individual-item lexical decision protocol. On each trial participant's press a "Yes" key if the presented item is a legitimate English word, and a "No" key otherwise. On each trial of the 2-word version of the task, letter strings were presented two at a time, one on top of the other. Participants pressed the "Yes" key if both the presented letter strings were words, and the "No" key otherwise. Finally, the 4-word condition presented four vertically stacked letter strings. Participants pressed the "Yes" key if all four letter-strings were words, and the "No" key otherwise.

If the decision chains combine additively, the means and the variances of the 1-word, 2-word, and 4-word conditions should double, or add, as a function of the condition progression. By contrast, multiplication in the linear domain equals addition in the logarithmic domain. Thus, if the task relies on multiplicative relations, then differences in the logarithm of the mean response times should better approximate additivity and the Gaussian distribution, compared to the untransformed variables. If this relation holds on a logarithmic scale, it is striking evidence for multiplicative interaction because the sum of logarithms equals multiplication of the untransformed values. Furthermore, additivity predicts the difference in the *variance* between the 1- and 2-word distributions is statistically equal to the difference between the variance of sums of two 1-word responses and the 2-word variances, likewise for sums of four 1-word responses and the 4-word variances. This is because the variance of the sum of two independent random variables equals the sum of their variances. The purpose of the experiment and statistical tests is to determine if cognitive performance is best characterized by additive or multiplicative dynamics. The research hypothesis is that cognitive performance is more consistent with multiplicative dynamics than with additive dynamics.

## Scientific Methods

*Participants*

One-hundred and eighty introductory psychology students volunteered to participate as one of several options for fulfilling and outside of class research requirement. All were native English speakers with normal or corrected to normal vision. Participants were assigned, at random, to the 1-word, 2-word, or 4-word lexical decision conditions. Based on condition, participants were randomly assigned to complete one of four potential word lists in the 1-word condition, and one of two lists in the 2-word condition. All participants assigned to the 4-word condition saw the same list. The data collection activities were approved and monitored by the institutional review board.

*Protocol*

*Stimuli & Design.* The key experimental word stimuli were selected from a source list of 3859 four- and five-letter words. This list included 2145 monosyllabic words used previously by Spieler and Balota (1997) plus 1714 additional four- and five-letter words that were not in the Spieler and Balota corpus.[10] The latter words were multisyllabic, morphographic variants of base words, homographs, or had vowel onsets; we included them to make our source list more broadly representative of the four- and five-letter words in natural discourse. The source list spanned a frequency range of 10,595-0 per million ($M$ = 65.74, $SD$ = 317.54)[11]. From this list, we randomly selected a candidate set of 548 words, drawn without replacement.

Next, we ran a pilot lexical decision study that included all 548 words presented to 13 participants using the procedure of the 1-word condition, without feedback. All word items that produced error rates greater than 25% were eliminated, leaving 455 words. Five more words were eliminated that did not appear in Kučera and Francis (1967), leaving 450 candidate words. Finally, we rank ordered the words based on frequency counts and eliminated the 50 most frequent words, leaving 400 words with frequency counts that ranged from 145 to 1 per million ($M$ = 22.06, $SD$ = 29.26).

All participants in 1-word, 2-word, and 4-word conditions were each presented with 100 word-trials. To equate stimulus properties across these conditions, the list of 400 target words were divided randomly into four lists of 100 words, presented to equal numbers of participants in the 1-word lexical decision condition. The 1- and 4-word conditions used all the same words. For the 2-word condition, each of the lists of 100 word-trials was collapsed into a list of 50 pairs by randomly pairing items, without replacement. The resulting four lists of fifty 2-word trials were then randomly selected, two at a time, and combined into two lists, of 100 trials each. These two lists were presented to equal numbers of participants in the 2-word condition. To construct the single list for the 4-word condition, we began with the same four lists of 50 paired words that were used to construct 2-word lists. Each list of 50 paired words was collapsed into a list of twenty-five 4-word quartets by randomly pairing combinations of 2-word trials without replacement. Then the four lists of twenty-five 4-word quartets, each, were combined into a single list of one-hundred 4-word trials.

A second, separate set of 548 words was selected at random from the source list and used to make pseudowords. A candidate set of pronounceable pseudowords was constructed by randomly switching onsets and word bodies among the set. After the switch, any fortuitously constructed pseudo-homophones, or words, were changed into pseudowords by rearranging letters. The changes always preserved the initial letter, length and pseudo-morphological structure (e.g., plural or past tense endings). Words with vowel onsets were changed to pseudowords by substituting a single interior letter. The candidate set of pseudowords were also included in the pilot lexical decision study. All pseudowords with error rates less than 15% and greater than 25% were eliminated, which left 100 pseudowords.

The 2-item nonword trials presented a nonword and a word. The 4-item nonword trials presented a nonword with three words. The words that accompanied the pseudoword targets in these conditions were randomly chosen, without replacement, from the items in the source list not previously selected. Stimuli for eighty practice trials (40 *word* and 40 *nonword*) trials were randomly selected from items rejected from the pilot study. These trials presented items that were, no doubt, more difficult to judge than experimental trials, but the level of difficulty, estimated from pilot error rates, was equated across the three experimental conditions.

*Procedure.* On each trial, in each condition, a fixation stimulus (+) was visible for 200 ms, and then disappeared for 200 ms, after which a target appeared. (One item targets appeared in the 1-word condition, two item targets in the 2-word condition, or four item targets in the 4-word condition). Targets remained visible until a participant responded. Participants responded by pressing a "Yes" key for all word only trials, and "No" key otherwise. Responses ended the trial, and triggered the fixation stimulus, which appeared after a 200 ms inter-stimulus-interval.

The three experimental conditions were the three different versions of lexical decision. Each presented 100 word-trials and 100 nonword-trials. The 1-word condition presented individual words, and the nonword trials presented individual pseudowords, centered with respect to the location of the previously presented fixation stimulus. The 2-word condition presented items in pairs, one above the other, and centered at the location of the fixation stimulus. Each two-item nonword trial presented a word and a pseudoword, and the position of the pseudoword was split equally, and randomly, across trials, between top and bottom. The 4-word condition presented quartets of words stacked vertically, one above the other, and centered about the location of the fixation stimulus. A pilot study indicated stacked quartets yield slightly faster response times than quartets presented in a 2X2 matrix. The 100 four-item nonword trials each presented a single pseudoword with three words. Pseudowords appeared equally often in each of the four possible positions, randomized across trials.

Participants were instructed to maintain error rates below 10%. Error responses triggered a feedback display that presented, for two seconds, a participant's overall error rate, to that point in the experiment. Each target appeared in the center of a computer monitor controlled by ERTS software, running on an IBM compatible PC. Response times were measured reliably to within one millisecond. Each participant completed 80 randomly ordered practice trials after which a feedback screen presented their mean, correct, response time, for the practice trials, and their overall error rate. Participants initiated the experimental trials by pressing a button that cleared the feedback screen. Following that, participants completed 200 randomly ordered experimental trials.

**Results**

All statistical tests used an *a priori* statistical significance level of $p \leq 0.05$. First, we conducted standard means analyses to test whether performance differed among the four wordlists in the 1-word condition, or the two wordlists in the 2-word condition. These standard one-way ANOVAs included all correct response times greater than 200 ms. Neither analysis produced statistically reliable differences (both $F$s < 1), and we collapsed across list factors in the remaining analyses.

*Test for Additive vs. Multiplicative Interaction*

The working hypothesis was that much of the variability in response times emerges because of multiplicative interaction among random variables. An important implication is that a logarithmic transformation of the raw data should render the statistical behavior of response time more in-line with the expectations of a variable that is distributed as the tame, garden variety Gaussian distribution.

The first analysis concerns the mean of the logarithmically transformed response times. The subject means of the logarithmically transformed correct response times were 6.47 ($SD$ = .16), 7.12 ($SD$ = .16) and 7.76 ($SD$ = .22) for the 1-, 2-, and 4-word tasks, respectively. The mean difference between the 1- and 2-word means was .645 and the mean difference between the 2- and 4-word means was also .647. Both differences were statistically significant $r^2 = .81$, $F(1,118) = 493.32$, $p < .05$ and $r^2 = .74$, $F(1,118) = 337.64$, $p < .05$. Thus, only after a logarithmic transformation did the sampling distribution of subject means reasonably conform to CLT expectations. Moreover, the mean differences between 2-word and 1-word, as well as the 4-word and 2-word conditions were statistically equivalent on the logarithmic scale. From the perspective of mean of correct responses, the 1-, 2-, and 4-word trials appear to combine multiplicatively, rather than additively.

An additional test concerns the inherent patterns of variability in the raw distributions of response time themselves. Historically, cognitive psychology assumed cognitive performances were governed by a set of component processes. Response time is a random variable. Response times are assumed to represent the overall finishing times of a set of component cognitive process that intervene between the time a stimulus is presented, and a response is recorded in a cognitive task.

The variance of the sum of two independent random variables equals the sum of their own variances. If one stacks two or four lexical decisions on top of each other, is the variability of the 2-word task approximately equal to the variability of a random variable that is formed by summing individual, randomly selected 1-word times? This test entails a very conservative perspective on the time required to complete a lexical decision because it implies there are no time savings in deciding the lexical statuses of two simultaneously presented items over two individually presented items. Thus, the test works in favor of the additive hypothesis.

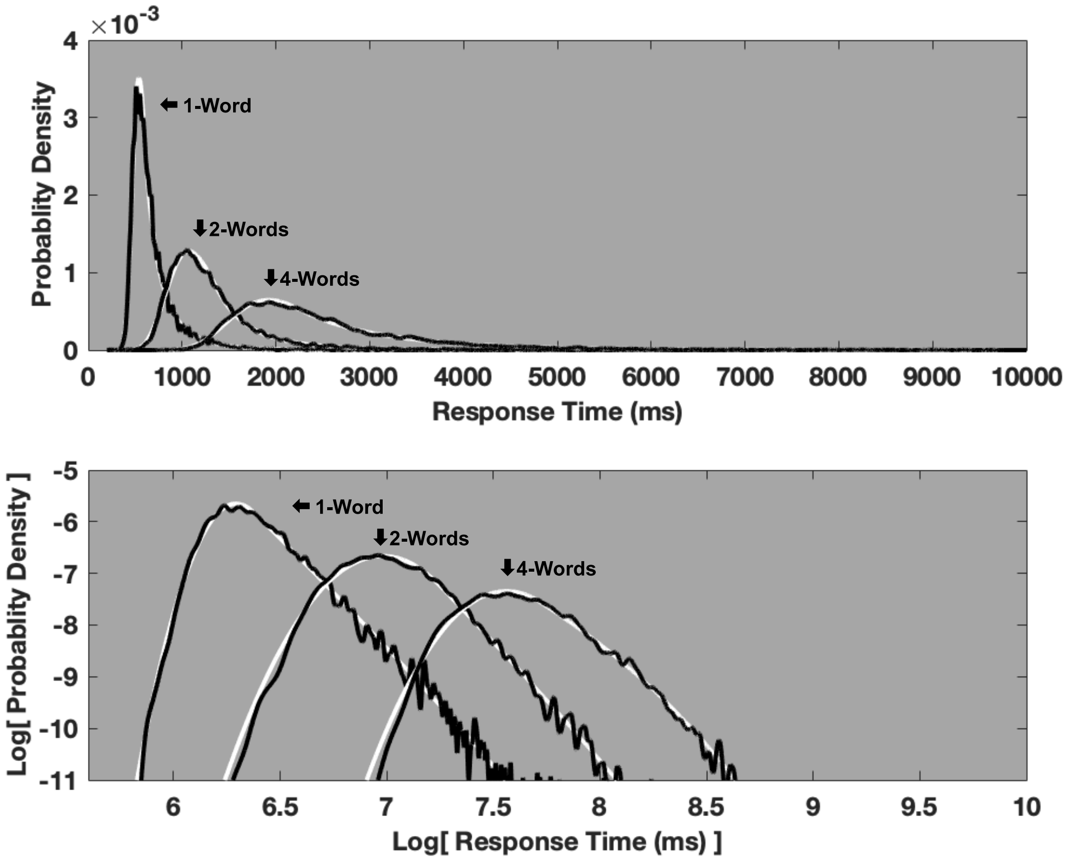


**Figure 1.** The upper plot depicts the kernel-smoothed empirical 1-, 2- and 4-word probability density functions from left to right as black lines. A lognormal power-law mixture distribution was fit to, and plotted behind each density function as white lines. The lower plot depicts the same empirical and model distributions, now on double-logarithmic axes. Notably, the distributions tend to maintain a linear decay in their right tails, consistent with power law scaling. The pattern persists even as the item count increases, which violates the expectations given by the CLT for additive combination of random variables.

Bootstrapping techniques simulated the null hypothesis of additive combinations. We arranged all 5667 correct "word" response times from the 1-word task that were greater than 200 ms into a single column vector. We then randomly sampled, with replacement, from the 5667 response times. We added that sample to an additional 5667 response time sample, from the same distribution, and computed the variance of a new variable that represented the sum of two 1-word lexical decision times. We repeated the procedure 1,000 times and recorded the value of the variance statistic resulting from the operation. The bootstrapped variances were then sorted in ascending order. The 950$^{th}$ largest variance (i.e., $p \leq 0.05$) was 1.71*E+5, which corresponds to the 95$^{th}$ percentile for the variance that results from the summation operation. The observed variance for the 2-word task was 3.18*E+5, which is well above the 99$^{th}$ percentile for the bootstrap summed variables. In fact, the 2-word variance is about twice as large as the average variance that resulted from the summation operation. A similar analysis combined four randomly selected 1-word response times to simulate the variance of the successive addition of 4 items, and the variance of the 4-word condition. Here, the 950$^{th}$ largest variance (i.e., $p \leq 0.05$) was 3.34* E+5, which corresponds to the 95$^{th}$ percentile for the variance that results from the summation operation. However, the observed 4-word variance larger, 1.38*E+6. Overall, the variance in the multiword conditions is larger than predicted if the durations of the separate word recognition acts are independent and additive.

**Discussion**

The log-transformed means of the 1- 2- and 4-word lexical decision tasks closely approximated equal-interval differences. The apparent additivity on a logarithmic scale is equivalent to multiplicative, or proportional differences across the conditions. Bootstrapping tests indicated the empirical 2-word, and 4-word variances were consistently larger than sums of two 1-word or four 1-word variances. A probability density function that represents a mixture of samples from a lognormal and an inverse power law distribution successfully approximated all three of the response time distributions.

This simple study can be interpreted as testing the fundamental assumption of additivity that is required of scientific hypotheses of modular cognitive architectures. Whatever the cognitive components that might participate in lexical decision, their presence should be consistently represented across distinct decision acts if their effects are additive. However, the successive decisions produced more variance than anticipated by additive dynamics. The ostensibly independent stacked decisions produced patterns of variability that better resembled multiplicative variability, or at least variability associated with strongly coordinated variables. Finally, a lognormal power-law model successfully approximated the three distributions. These mixture densities imply the presence of proportional scaling and nonlinear feedback dynamics that contradict the theoretical assumptions supplied by additive dynamics, the general linear model, and psychological theories that rely exclusively on reductive analysis.[12,13,14,15] Likewise, reports of long-range coordination, called 1/*f* noise, a form of temporal scaling, in trial-by-trial time-series of response times is consistent with a hypothesis that cognition does not reduce to the characteristic processing times anticipated by modularity.[8] Both scientists and practitioners are exploiting these discoveries to reframe long-standing puzzles in cognitive performance, psychotherapy, and the diagnosis of cognitive impairments.[15,17,18,19]

**Conclusions**

The manipulation, inspired by the renormalization group operation in statistical physics, tested the assumptions of additivity and independence required by the modularity hypothesis. Essentially, independent versions of the same judgment were repeated several times to "course-grain" a given cognitive act across time.[16] The persistent skew in response times taken from sequences of distinct decisions are more consistent with interdependence and multiplicative interaction than with statistical independence and additivity. The implication is that it may be more scientifically fruitful to understand cognitive performance as a problem of pattern formation in a complex system, rather than as a linearly decomposable information processing system.

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https://homepages.uc.edu/~holdenjn/rsrch/index_r.html

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